

## **An examination of topological spaces' fixed point theory and its uses**

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### **Abstract:**

One of the most important branches of contemporary mathematics is fixed point theory, which has wide-ranging effects on both the pure and practical sciences. The study of fixed points in different structures has yielded profound insights into analysis, topology, optimization, and nonlinear functional equations. A fixed point of a mapping is a point that stays invariant under the action of the mapping. We provide a thorough analysis of fixed point theory in the context of topological spaces in this paper. We examine traditional findings like the fixed point theorems of Brouwer and Schauder as well as more recent extensions in metric, normed, and topologically ambiguous circumstances. The importance of contractive conditions, continuity, and compactness in proving the existence and uniqueness of fixed points is emphasized. We also emphasize the use of fixed point theory in dynamical systems, game theory, mathematical economics, and differential equations. Along with discussing possible study avenues, such as fixed points in Hilbert spaces, nonlinear mappings, and their relationships to contemporary computer techniques, the work not only synthesizes previous findings.

**Keywords:** Function , Topology , Topology space , Set theory , Fixed Point Theory , Mapping

## **1. Introduction**

Topology is a really interesting branch of mathematics, and it forms in analysis, geometry, and algebraic topology.

Point set topology is traditional part of topology. It deals with ideas like continuity, homeomorphism of functions, and the compactness and connectedness of topological spaces. Examples of topics in topology include sets, mappings (which are functions), and topological spaces. Topology is also the branch that links geometry and algebra.

A family of sets with specific properties used to define a topological space, which is a basic concept in topology. Homeomorphisms are especially important; they are defined as continuous functions that have a continuous inverse. Topology includes several subfields, such as point-set topology, which establishes the foundation of topology and explores concepts related to topological spaces; algebraic topology, which generally measures how connected something is using algebraic tools like homology groups and homology; and geometric topology, which mainly studies manifolds and their embeddings in other manifolds. In recent years, many of the new ideas in mathematics have originated in topology from geometrical images, which were then formalized and applied to more algebraic areas.

The following sets, namely GRW-open sets, GRW-locally closed sets, GRWlc\*, GRWlc, and semi-GRW-closed sets, are being studied using GRW-closed sets as well as other types of continuity. GRW-closed sets in ditopological spaces are currently a main focus of research. In ditopological spaces, additional concepts such as GRW-continuity, GRW-irresolute, and GRW-homeomorphism are defined and explored. Future studies could further examine the properties of GRW-homeomorphisms in both topological and ditopological spaces. In ditopological spaces, we can introduce GRWc-homeomorphism, connectedness, and compactness. Other spaces such as ideal topological spaces, grill topological spaces, digital topological spaces, digital lines, and fuzzy topological spaces can also be used to study GRW-closed sets.

Let's now explore the various applications of topology and understand its impact. Applications to Digital Image Processing: Today, digital images are a major way to share and work with visual information. Examples include photos from a digital camera, text in a book, artwork, and graphics. Digital image processing involves creating, storing, manipulating, and showing these images. In each step of this process, topological ideas and tools are used to address different challenges. Digital topology plays a central role here. It is based on a digital plane, which is formed by combining two digital lines. A digital line is simply the set of all integers,  $\mathbb{Z}$ . For each odd integer  $n$ , the basic element is  $B(n) = \{n\}$ , and these integers are called pixels. So, each pixel is considered an open set in digital topology. Digital topology studies how topological properties affect the way digital images are displayed. The digital plane in this context is the topological space  $\mathbb{Z} \times \mathbb{Z}$ . A visible screen is a part of this digital plane that includes all the open points. For every  $(m, n)$  in  $\mathbb{Z} \times \mathbb{Z}$ , this applies.  $\mathbb{Z}$ , basis element is given as:

$$B(m, n) = \begin{cases} (m, n) & \text{if } m \text{ and } n \text{ are odd} \\ (m + a, n) \{a = -1, 0, 1\} & \text{if } m \text{ is even and } n \text{ is odd} \\ (m, n + b) \{b = -1, 0, 1\} & \text{if } m \text{ is odd and } n \text{ is even} \\ (m + a, n + b) \{a, b = -1, 0, 1\} & \text{if } m, n \text{ are even} \end{cases}$$

Even though a digital plane can also be made by dividing the plane of real numbers with standard topology, the key focus in digital image processing is to study the features of an object from its digital image. In 1979, a paper by Azriel Rosenfeld titled "Digital Topology" introduced the first work that looked at how topological ideas like connectedness and continuity work in a digital setting. Later on, these ideas were refined, and special topological spaces were created to model digital images, allowing topological concepts to be used directly in digital environments. Since the real world is seen as connected and continuous, it's necessary to use tools that turn images into digital form while keeping a connected structure to match the real-world relationships. The digital plane, which is made up of points in a grid like  $\mathbb{Z} \times \mathbb{Z}$ , has open, totally disconnected, and dense visible structures along with hidden structures that provide connectivity. This makes it the right model for digital image processing. So, the role of topology in digital image processing is just as important as it is in general topology. Application to Robotics: Topology and physics have a strong connection. To explore complex topics in other areas, you need a deep understanding of topology. In physics, the first step is to study the configuration space, which acts as a topological space. To do this, we need to track variables related to the positions and arrangements of objects. For example, when working with a robot arm, we need to monitor the movement of its different parts. The configuration space helps us track these variables effectively. It also helps in studying other aspects like momentum and velocity, leading to the concept of phase space. Topology is also very useful for studying functions. One important map is the forward kinematics map, which is essential in designing movement for robots and other machines. This map helps identify problematic or difficult arrangements in a mechanism. In robotics, there's a specific point at the end of

the arm that performs tasks like picking up objects, drilling, or painting. This point is called the end effectors.

## 2. GENERALIZED $\alpha$ b-CLOSED SETS :

**Theorem 1.1 :** Let  $\mathcal{A}, \mathcal{B}, \mathcal{S}, \mathcal{T}$  be self mapping of a fuzzy metric space  $(\mathcal{X}, \mathcal{M}, *)$  satisfying conditions  $\mathcal{A}(\mathcal{X}) \subset \mathcal{T}(\mathcal{X})$  and  $\mathcal{B}(\mathcal{X}) \subset \mathcal{S}(\mathcal{X})$  ( 3.2.1)

$$M(\mathcal{A}x, \mathcal{B}y, t) \geq \emptyset(\min\{\mathcal{S}x, \mathcal{T}y, t\}, M(\mathcal{S}x, \mathcal{A}x, t), M(\mathcal{B}y, \mathcal{T}y, t)) \quad (3.2.2)$$

Where  $x, y \in \mathcal{X}$  and  $\emptyset: [0,1] \rightarrow [0,1]$   $\emptyset(s) \geq s$ , where  $0 < s < 1$

A arbitrary point  $x_0 \in \mathcal{X}$  then the sequence  $\{y_n\}$  defined by

$$y_{2n} = \mathcal{T}x_{2n+1} = \mathcal{A}x_{2n},$$

$$y_{2n+1} = \mathcal{S}x_{2n+1} = \mathcal{B}x_{2n+1} \forall n = 0, 1, 2, \dots \quad (3.2.3)$$

Is a Cauchy sequence in  $\mathcal{X}$

Proof: for  $t > 0$

$$M(y_{2n}, y_{2n+1}, t) = M(\mathcal{A}x_{2n}, \mathcal{B}x_{2n+1}, t)$$

$$M(y_{2n}, y_{2n+1}, t) \geq \emptyset(\min\{M(\mathcal{S}x_{2n}, \mathcal{T}x_{2n+1}, t), M(\mathcal{S}x_{2n}, \mathcal{A}x_{2n}, t), M(\mathcal{B}x_{2n+1}, \mathcal{T}x_{2n+1}, t)\})$$

$$M(y_{2n}, y_{2n+1}, t) = \emptyset(\min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n-1}, y_{2n}, t), M(\mathcal{B}x_{2n+1}, \mathcal{T}x_{2n+1}, t)\})$$

Case first  $M(y_{2n}, y_{2n+1}, t) > M(y_{2n-1}, y_{2n}, t)$

if  $M(y_{2n-1}, y_{2n}, t) < M(y_{2n}, y_{2n+1}, t)$

Case two  $M(y_{2n}, y_{2n+1}, t) > M(y_{2n}, y_{2n+1}, t)$

if  $M(y_{2n-1}, y_{2n}, t) \geq M(y_{2n}, y_{2n+1}, t) \dots \dots \dots (3.2.4)$

As  $\emptyset(s) \geq s$ , where  $0 < s < 1$

Thus  $M(y_{2n}, y_{2n+1}, t) \geq 0$  is an  $[0,1]$  and limit  $l \leq 1$  we declare  $l = 1$

Or then  $l < 1$  which on letting  $n \rightarrow \infty$  in  $\dots \dots \dots (3.2.4)$

We get  $l \geq \emptyset(l) > l$  a contradiction our hypothesis  $l = 1$

So  $M(y_{2n+1}, y_{2n+2}, t), n \geq 0$  where  $n \in \mathcal{N}$

Is a sequence of positive number  $[0,1]$  which tends to  $l = 1, n \in \mathcal{N}$

$$M(y_n, y_{n+1}, t) > M(y_{n-1}, y_n, t)$$

And  $\lim_{n \rightarrow \infty} M(y_n, y_{n+1}, t) = 1$

Now for any positive integer  $h$

$$M(y_n, y_{n+h}, t) > M(y_{n-1}, y_n, \frac{t}{h}) * \dots * M(y_{n+h-1}, y_{n+h}, \frac{t}{h})$$

$\therefore \lim_{n \rightarrow \infty} M(y_n, y_{n+1}, t) = 1$  for  $t > 0$

It follows that  $\lim_{n \rightarrow \infty} M(y_n, y_{n+h}, t) \geq 1 * 1 \dots * 1 = 1$

Which shows that the sequence  $\{y_n\}$  is a Cauchy sequence in  $\mathcal{X}$ .

**Theorem 1:** Let  $(X, d)$  and  $(Y, e)$  be fuzzy set. If  $T$  is a mapping from  $X$  into  $Y$  and  $S$  is a mapping from  $Y$  into  $X$  satisfying the following conditions

$$e(t_x, Ts_y) \leq c_1 \cdot \max\{d(x, s_y), e(y, t_x), e(y, Ts_y), d(x, St_x), d(s_y, St_x)\} \text{ --- (1)}$$

$$d(s_y, St_x) \leq c_2 \cdot \max\{e(y, t_x), d(x, s_y), d(x, St_x), e(t_x, Ts_y), e(y, Ts_y)\} \text{ --- (2)}$$

for all  $x$  in  $X$  and  $y$  in  $Y$  where  $0 \leq c_1 < 1$  and  $0 \leq c_2 < 1$ ,

then  $ST$  has a unique fixed point  $z$  in  $X$  and  $TS$  has a unique fixed point  $w$  in  $Y$ .

Further  $t_z = w$  and  $s_w = z$ .

**Proof.** Let  $x_0$  be an arbitrary point in  $X$ . Define a sequence  $\{x_n\}$  in  $X$  and a sequence  $\{y_n\}$  in  $Y$ , as follows:

$$x_n = (ST)nx_0, y_n = T(x_{n-1}) \text{ for } n = 1, 2, \dots$$

$$\text{We have } d(x_n, x_{n-1}) = d(Sy_n, STx_n) \leq c_2.$$

$$\max\{e(y_n, Tx_n), d(x_n, Sy_n), d(x_n, STx_n), e(Tx_n, TSy_n), e(y_n, TSy_n)\} = c_2.$$

$$\max\{e(y_n, y_{n+1}), d(x_n, x_n), d(x_n, x_{n-1}), e(y_{n+1}, y_{n+1}),$$

$$e(y_n, y_{n+1})\} \leq c_2 \cdot e(y_n, y_{n+1}) \text{ --- (3)}$$

Now

$$e(y_n, y_{n+1}) = e(Tx_{n-1}, Tx_n) = e(Tx_{n-1}, TSy_n) \leq c_1.$$

$$\max\{d(x_{n-1}, Sy_n), e(y_n, Tx_{n-1}), e(y_n, TSy_n),$$

$$d(x_{n-1}, STx_{n-1}), d(Sy_n, STx_{n-1})\} \leq c_1 \cdot d(x_{n-1}, x_n) \text{ --- (4)}$$

Hence using inequalities (3) and (4),

$$\text{we have } d(x_n, x_{n-1}) \leq c_1 c_2 \cdot d(x_{n-1}, x_n) \therefore (c_1 c_2)^n \cdot d(x_0, x_1) \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$(\text{since } 0 \leq c_1 c_2 < e(Tz, w) \text{ (since } c_1 c_2 < 1)),$$

Which is a contradiction.

Thus  $Tz = w$ . To prove that  $Sw = z$ .

$$\text{Suppose that } Sw \neq z. d(Sw, z) = \lim_{n \rightarrow \infty} d(Sw, STx_n) \leq \lim_{n \rightarrow \infty} c_2.$$

$$\max\{e(w, Tx_n), d(x_n, Sw), d(x_n, STx_n), e(Tx_n, TSw), e(y_n, TSx_n)\} \leq c_2.$$

$$e(w, TSw) \text{ Now } e(w, TSw) = \lim_{n \rightarrow \infty} e(Tx_n, TSw) \leq \lim_{n \rightarrow \infty} c_1.$$

$$\max\{d(x_n, Sw), e(w, Tx_n), e(w, TSw), e(Tx_n, TSw), e(y_n, TSx_n)\} \leq c_1 \cdot d(Sw, z)$$

$$\text{Hence } d(Sw, z) \leq c_1 c_2 \cdot d(Sw, z) < d(Sw, z) \text{ (since } c_1 c_2 < 1)$$

Which is a contradiction. Thus  $Sw = z$ .

We have  $STz = Sw = z$  and  $TSw = Tz = w$ .

Thus the point  $z$  is a fixed point of  $ST$  in  $X$  and the point  $w$  is a fixed point of  $TS$  in  $Y$ .

Uniqueness: Let  $z' \neq z$  be another fixed point of  $ST$  in  $X$ .

We have

$$d(z, z') = d(Sw, STz') \leq c_2 \cdot \max\{e(w, Tz'),$$

$$d(z', Sw), d(z', STz'), e(Tz', TSw), e(w, TSw)\} \leq c_2.$$

$$e(Tz', w) \text{ Now } e(Tz', w) = e(Tz', TSw) \leq c_1.$$

$$\max\{d(z', Sw), e(w, Tz'), e(z', TSz'), e(Tx_n, TSw), e(yn, TSx_n)\} \leq c_1.$$

$$d(z, z') \text{ Hence } d(z, z') \leq c_1 c_2 \cdot d(z, z') < d(z, z') \text{ (since } c_1 c_2 < 1)$$

Which is a contradiction.

Thus  $z = z'$ .

So the point  $z$  is a unique fixed point  $z$  of  $ST$ . Similarly, we prove the point  $w$  is also a unique point of  $TS$ .

Now consider a non-empty set  $X$ .

A crisp subset  $A$  of  $X$  is defined by specifying which elements of  $X$  are in  $A$ . In other words, a crisp subset  $A$  of  $X$  can be studied using a characteristic function that assigns to each element of  $X$  a value of either 1 (if it belongs to  $A$ ) or 0 (if it does not belong to  $A$ ). Therefore, any element of  $X$  either belongs to subset  $A$  or does not belong to subset  $A$ .

However, in real-life situations, there are many instances where it is not possible to clearly state whether an element of  $X$  belongs to subset  $A$  or not.

For example, consider  $X$  as the collection of students in a particular class. Since every student has some level of intelligence that can be graded, it is not possible to clearly define the intelligence of a student as a crisp subset of  $X$ .

**Theorem 2: [2]** Let  $S$  and  $t$  be the using the mapping then shoe that the matrix space is used to the  $S(X) \subset X, T(X) \subset X$

$$d(s_x, t_y) \leq \left\{ \max \left\{ \frac{d(x, s_x)d(y, s_x) + d(y, t_y)}{1 + d(s_x, t_y)}, \right. \right. \\ \left. \frac{d(y, s_x)d(x, t_y) + d(x, y)d(s_x, y)}{d(s_x, t_y) + d(s_x, y)} \right\}, \\ \frac{d(x, s_x)d(y, s_x) + d(x, y)d(s_x, t_y)}{d(y, t_y) + d(y, s_x)}, \\ \left. \frac{d(y, s_x)d(x, t_y) + d(x, t_y)d(y, s_x)}{d(y, t_y) + d(y, s_x)} \right\}$$

We mostly expand and enhance theorem in this section. By the way, we simplify the theorem equation (2)

proof. First, we present a different straightforward proof of Theorem (2).

Praveen and colleagues (2014) to provide evidence of the sequence of functions that include H-functions. The proof was accomplished with the help of various operational approaches. He presented several generating relations along with a few formulas that were associated with finite summation. In addition, he discussed the significance of the H-function in terms of sequencing. In general, the H-function appears to be a type of large number sequence, but in some special cases, it also involves smaller polynomial functions. He repeated his statement regarding the linear differential equation, which he said is easy to compute. This might take the form of a new operator-valued function, which we can refer to simply as a function and its solution. We can obtain this by performing an inverse function on it. A.

Jyotindra, and others (2013) presented techniques in operational research, also known as operational calculus. Here, he discussed some functions of sequences that involve polynomials. The operational techniques refer to the way of showing the different steps of calculus, as well as its derivations and integrations. All of this combined is known as operational research. By using this calculus, we can analyze problems. Differentiation, which plays a key role in the operational method and which we often use when solving polynomial equations, is one of the techniques we generally apply. Its abbreviation is D, which is the same as  $\frac{d}{dx}$ , and it operates on functions. We also have a new operator-valued function F(D). It has the same value based on its established role.

### 3. GENERALIZED D b-CONTINUOUS MAPS IN FUZZY TOPOLOGICAL SPACES :-

Researchers have carried out studies on fuzzy topological spaces, focusing on fuzzy generalized  $\tilde{A}$ -closed sets, generalized  $\tilde{A}$ -open maps, and homeomorphisms. The ideas presented aim to inspire the application of these new concepts across various areas of topology. The work could be expanded to explore analytical topics such as connectedness, compactness, and convergence, which are fundamental in topology. It is recommended that this study be introduced and examined within a minimal structure. This approach could also be applied in Digital Topology. Additionally, the new ideas of fuzzy bi-topological spaces can be explored for these sets. Furthermore, the breakdown and analysis of generalized b-closed and generalized b-open sets can be investigated.

**Definition 2.1:** Given a Cone Banach Space  $(M, \|\cdot\|)$ , two self mappings  $\emptyset$  and  $\mu$  on the space satisfy the property (E.A.) for a sequence  $\{u_p\}$  in such a way that:

$$\begin{aligned} &= \lim_{p \rightarrow \infty} \emptyset u_p \\ &= \lim_{p \rightarrow \infty} \mu u_p, \text{ for some } t \in M \end{aligned}$$

**Examples 2.2:** Let  $M = [0, 1]$ , Define  $\emptyset, \psi: M \rightarrow M$  such that

$\emptyset(u) = 2 - u$  and  $\psi(u) = \frac{3-u}{2}$  Consider a sequence

$u_p = \frac{1+1}{p}$  we have,  $\lim_{p \rightarrow \infty} \emptyset(u_p)$

$$= \frac{2-1}{1} p = 1 \text{ and } \lim_{p \rightarrow \infty} (u_p)$$

$$= \frac{3}{2} - \frac{1}{2} = \frac{2}{2} \quad p = 1 \quad \lim_{p \rightarrow \infty} \emptyset(u_p)$$

$$= \lim_{p \rightarrow \infty} (u_p) = 1 \text{ Clearly } \emptyset \text{ and } \psi \text{ satisfy property (E.A.)}$$

**Example 2.3:** Let  $M = [0, 1]$ , Define  $F, \psi: M \rightarrow M$  such that

$\emptyset(u) = 1 - u$  and  $\psi(u) = u$  Consider a sequence

$$\begin{aligned} up = \frac{1-1}{p} \text{ we have, } \lim_{p \rightarrow \infty} \emptyset(u_p) \\ = \frac{1-1}{1-p} = 0 \text{ and } \lim_{p \rightarrow \infty} (u_p) \\ = \frac{1-1}{p} = 1 \lim_{p \rightarrow \infty} \emptyset(u_p) \neq \lim_{p \rightarrow \infty} \psi(u_p) \end{aligned}$$

Clearly  $\emptyset$  and  $\psi$  do not satisfy property (E.A.)

**Theorem 2.4.:** Four self-mappings  $F, G, H$  and  $L$  be defined on Cone Banach Space  $(\mathbb{M}, \|\cdot\|)$  with the norm  $\|u\| = d(u, 0)$  satisfying the conditions

$$\|Hu - Lv\| \leq a\|Fu - Hu\| + \|Fu - Lv\| + c\|Gv - Lv\| \dots (1)$$

for all  $u, v \in \mathbb{M}$  and  $a, b, c \geq 0$ ,  $a + 2b + c < 1$ .

- (i)  $F(\mathbb{M}) \subseteq G(\mathbb{M})$  and  $H(\mathbb{M}) \subseteq L(\mathbb{M})$
- (ii)  $(F, H)$  and  $(G, L)$  are weakly compatible.
- (iii) One of pair  $(F, H)$  and  $(G, L)$  satisfies property (E.A.) Then  $F, G, H$  and  $L$  have a unique common fixed point

**Proof:** Let  $(G, L)$  satisfy property (E.A.) then there exist a sequence  $\{u_p\}$  in (1)  $\mathbb{M}$  such that

$$\lim_{p \rightarrow \infty} L\{u_p\} = \lim_{p \rightarrow \infty} G\{u_p\}$$

$= t$  for some  $t \in \mathbb{M}$  Since  $(\mathbb{M}) \subseteq (\mathbb{M})$  then  $\exists$  a sequence  $\{v_p\}$  in  $\mathbb{M}$  such that

$$L\{u_p\} = \{v_p\} \text{ Hence, } \lim_{p \rightarrow \infty} \{v_p\} = t, \text{ We claim that}$$

$$\lim_{p \rightarrow \infty} H\{v_p\} = t$$

on the contradiction, we put  $u = v_p$  and  $v = u_p$  in (1)

$$\text{we have } \|Hv_p - Lu_p\| \leq \|Fv_p - Hv_p\| + \|Fu_p - Lu_p\| + \|Gu_p - Lu_p\|$$

From the above condition we get,  $\|$

$$\|Hv_p - Lu_p\| \leq a\|Fv_p - Hv_p\| + b \cdot 0 + c\|Gu_p - Lu_p\| \text{ Now taking}$$

$$\text{limit } p \rightarrow \infty \|Hv_p - t\| \leq a\|t - Hv_p\| + c\|t - t\| (1 - a)\|Hv_p - t\| \leq 0$$

$$\text{But } (1 - a) \neq 0 \text{ then } \lim_{p \rightarrow \infty} \{v_p\}$$

$$= t \text{ Hence } \lim_{p \rightarrow \infty} \{v_p\}$$

$$= \lim_{p \rightarrow \infty} \{v_p\}$$

$= t$  Suppose first that,  $(\mathbb{M})$  is complete subspace of  $\mathbb{M}$  then

$$t = (w) \text{ for some } w \in \mathbb{M} \text{ then } \lim_{p \rightarrow \infty} \{u_p\}$$

$$= \lim_{p \rightarrow \infty} \{v_p\}$$

$$= \lim_{p \rightarrow \infty} \{u_p\}$$

$$= \lim_{p \rightarrow \infty} \{v_p\} = t$$

$= (w)$  We claim that,

$Hw = Fw$ , on the contradiction we put  $u = w$  and  $v = u_p$  in (1)  $\parallel$

$$\|Hw - Lu_p\| \leq \|Fw - Hw\| + \|Fw - Lu_p\| + \|Gu_p - Lu_p\|$$

Letting  $p \rightarrow \infty$  and using above condition.

#### 4. Conclusion

In topological spaces, the outcome of fixed point theory is promising and bright. The need for generic, robust fixed point findings is growing as topological structures are increasingly used to model abstract and real-world systems. To ensure its relevance in contemporary mathematical science, this field must be advanced by a combination of in-depth theoretical study and useful multidisciplinary applications. The study of location is known as topology. While this particular area of mathematics emerged in the early 1900s, some of its concepts were well established previously. In 1736, Leonhard Euler published the first article on the Seven Bridges of Königsberg, which is regarded as the earliest instance of topology being used in practice.

Augustin-Louis Cauchy, Johann Benedict Listing, Ludwig Schläfli, Enrico Betti, and Bernhard Riemann are some of the other significant figures who have made contributions to the discipline. In 1847, Johann Benedict Listing's work "Vorstudien zur Topologie" is credited with coining the term "Topologie". In 1883, an article in the journal *Nature* used the English word "topology" for the first time to distinguish between "qualitative geometry" and regular geometry. Along with algebra and analysis, topology is generally regarded as one of the fundamental pillars of contemporary abstract mathematics. Initially, topological research was motivated by real-world issues., However, the emphasis changed to more abstract concepts after the field was formally established. Nonetheless, topology has recently had a significant influence on numerous other fields as well. Topology is being used by scientists and mathematicians to investigate and comprehend actual occurrences.

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